

# Primordial magnetic field limits from cosmological data

Tina Kahniashvili,<sup>1,2,3,\*</sup> Alexander G. Tevzadze,<sup>3,4,†</sup> Shiv K. Sethi,<sup>1,5,‡</sup> Kanhaiya Pandey,<sup>5</sup> and Bharat Ratra<sup>6,§</sup>

<sup>1</sup>*McWilliams Center for Cosmology and Department of Physics,  
Carnegie Mellon University, 5000 Forbes Ave, Pittsburgh, PA 15213, USA*

<sup>2</sup>*Department of Physics, Laurentian University, Ramsey Lake Road, Sudbury, ON P3E 2C, Canada*

<sup>3</sup>*Abastumani Astrophysical Observatory, Ilia State University, 2A Kazbegi Ave, Tbilisi, GE-0160, Georgia*

<sup>4</sup>*Faculty of Exact and Natural Sciences, Tbilisi State University, 1 Chavchavadze Ave, Tbilisi, GE-0128, Georgia*

<sup>5</sup>*Raman Research Institute, Sadashivanagar, Bangalore 560080, India*

<sup>6</sup>*Department of Physics, Kansas State University, 116 Cardwell Hall, Manhattan, KS 66506, USA*

(Dated: September 14, 2010 KSUPT-10/2)

We study limits on a primordial magnetic field arising from cosmological data, including that from big bang nucleosynthesis, cosmic microwave background polarization plane Faraday rotation limits, and large-scale structure formation. We show that the physically-relevant quantity is the value of the effective magnetic field, and limits on it are independent of how the magnetic field was generated.

PACS numbers: 98.70.Vc, 98.80.-k

## I. INTRODUCTION

There is much interest in the origin of the coherent part of the large-scale  $\mu\text{G}$  magnetic fields in galaxies [1].<sup>1</sup> A leading possible explanation is that these large-scale magnetic fields are the amplified remnants of a primordial seed magnetic field generated in the early Universe [5–7]. Such early magnetogenesis models include those in which seed magnetic field generation occurs during inflation or shortly thereafter, or during a cosmological phase transition (such as the electroweak or QCD transition). Clearly the strength of the seed magnetic field should be small enough so as to not generate a larger than observed cosmological anisotropy. Magnetic field energy density contributes to the relativistic (radiation) energy density and thus another requirement is that it not exceed the big bang nucleosynthesis (BBN) bound on the radiation energy density.

There are two main questions that need to be answered: (1) Are the amplitude and statistical properties of any of these seed magnetic fields such that, after amplification by a realistic model, they can explain the strengths and correlation lengths of the observed magnetic fields in large-scale structures (LSSs) such as galaxies? and, (2) Are any of the seed magnetic fields detectable through cosmological observations, such as cosmic microwave background (CMB) measurements<sup>2</sup> or

LSS observations? And, if yes, what are the observational constraints on such primordial magnetic fields?

In this paper we focus on the second question and consider two cosmological consequences of a primordial magnetic field, Faraday rotation of the CMB polarization plane and the effect on LSS formation. As two of the effects of a primordial magnetic field, these have been widely discussed in the literature. See Refs. [8–13] for discussions of magnetic field induced CMB anisotropies,<sup>3</sup> Refs. [15–17] for the Faraday rotation effect, and Refs. [18–22] for effects of a primordial magnetic field on LSS formation.

It has become conventional to derive the cosmological effects of a seed magnetic field by using a magnetic field spectral shape (parametrized by the spectral index  $n_B$ ) and the smoothed value of the magnetic field ( $B_\lambda$ ) at a given scale  $\lambda$  (which is usually taken to be 1 Mpc). We develop here a different and more correct formalism based on the effective magnetic field value that is determined by the total energy density of the magnetic field. As a striking consequence, we show that even an extremely small smoothed magnetic field of  $10^{-29}$  G at 1 Mpc, with the Batchelor spectral shape ( $n_B = 2$ ) at large scales, can leave detectable signatures in CMB or LSS statistics.<sup>4</sup>

We also show that the conventional approach based on the smoothed magnetic field results in some confusion when considering phase-transition generated mag-

\*Electronic address: tinatin@phys.ksu.edu

†Electronic address: aleko@tevza.org

‡Electronic address: sksethi@andrew.cmu.edu

§Electronic address: ratra@phys.ksu.edu

<sup>1</sup> On larger scales there have been two recent claims of an observed lower limit of order  $10^{-15}$  G on the intergalactic magnetic field [2, 3], as well as one claimed detection of a field of this strength [4]. Prior to these observations, the intergalactic magnetic field was observationally only limited to be smaller than a few nG.

<sup>2</sup> A cosmological magnetic field induces all three kinds of gravita-

tional perturbations, scalar, vector, and tensor; all three of which contribute to CMB temperature and polarization anisotropies.

<sup>3</sup> The effects of a homogeneous magnetic field on the scalar mode of CMB fluctuations, including the resulting non-Gaussianity of the CMB temperature map, are discussed in Refs. [14].

<sup>4</sup> This strong limit on the primordial magnetic field is the consequence of the BBN bound and the sharp shape of the magnetic field at large scales [23, 24]. The low efficiency of gravitational wave production by a cosmological magnetic field [24] results in a weaker limit on the seed field from the direct detection of the induced gravitational wave signal [25].

netic fields [26] with spectral shape sharper (on large scales) than the white noise spectrum (i.e. with  $n_B > 0$ ). In this case the total energy density of the magnetic field,  $\rho_B$ , is mainly concentrated on large wavenumbers (small length scales). In what follows we show that primordial magnetic field effects on cosmological scales are determined not by the amplitude of the magnetic field on these scales, but rather by the total energy density of the magnetic field.

Here we consider limits on a seed magnetic field from the observational constraint on the CMB polarization plane rotation angle and observational constraints on the formation of the first bound structures. Both tests give comparable limits on the effective magnetic field, ranging from  $10^{-9}$  G to  $10^{-7}$  G, depending on the spectral shape of the magnetic field. Note that these limits are of order of the BBN bound. The best limit on the seed magnetic field is for the scale-invariant case that can be generated during inflation [27, 28].

In a Universe with only scalar mode perturbations the CMB  $B$ -polarization signal vanishes. The CMB  $B$  polarization signal can arise from vector or tensor perturbations, and thus  $B$ -polarization detection based tests are powerful tools for probing non-standard cosmological models and the relic gravitational wave background. Since a cosmological magnetic field can source a CMB  $B$ -polarization signal, a crucial test to limit the magnetic field is based on CMB  $B$ -polarization measurements. In a separate paper we will address the cosmological magnetic field energy density limits that can result from this test.

The structure of our paper is as follows. In Sec. II we review magnetic field statistical properties. In Sec. III we examine the CMB polarization plane Faraday rotation effect and the resulting magnetic field limits. In Sec. IV we consider the influence of a magnetic field on LSS statistics and determine the resulting limits on the magnetic field. We discuss our results and conclude in Sec. V.

## II. MODELING THE PRIMORDIAL MAGNETIC FIELD

A stochastic Gaussian magnetic field is fully described by its two-point correlation function. For simplicity we consider here the case of the non-helical magnetic field for which the two-point correlation function in wavenumber space is [11]

$$\langle B_i^*(\mathbf{k}) B_j(\mathbf{k}') \rangle = (2\pi)^3 \delta^{(3)}(\mathbf{k} - \mathbf{k}') P_{ij}(\hat{\mathbf{k}}) P_B(k). \quad (1)$$

Here  $i$  and  $j$  are spatial indices,  $i, j \in (1, 2, 3)$ ,  $\hat{k}_i = k_i/k$  a unit wavevector,  $P_{ij}(\hat{\mathbf{k}}) = \delta_{ij} - \hat{k}_i \hat{k}_j$  the transverse plane projector,  $\delta^{(3)}(\mathbf{k} - \mathbf{k}')$  the Dirac delta function, and  $P_B(k)$

is the power spectrum of the magnetic field.<sup>5</sup>

We define the smoothed magnetic field  $B_\lambda$  through the mean-square magnetic field [11]

$$B_\lambda^2 = \langle \mathbf{B}(\mathbf{x}) \cdot \mathbf{B}(\mathbf{x}) \rangle_\lambda, \quad (3)$$

where the smoothing is done on a comoving length  $\lambda$  with a Gaussian smoothing kernel function  $\propto \exp[-x^2/\lambda^2]$ . Corresponding to the smoothing length  $\lambda$  is the smoothing wavenumber  $k_\lambda = 2\pi/\lambda$ . The power spectrum  $P_B(k)$  is assumed to be a simple power law on large scales,  $k < k_D$  (where  $k_D$  is the cutoff wavenumber),

$$P_B(k) = P_{B0} k^{n_B} = \frac{2\pi^2 \lambda^3 B_\lambda^2}{\Gamma(n_B/2 + 3/2)} (\lambda k)^{n_B}, \quad (4)$$

and assumed to vanish on small scales where  $k > k_D$ .

The energy density of the magnetic field is [13]

$$\rho_B(\eta_0) = \frac{B_\lambda^2 (k_D \lambda)^{n_B+3}}{8\pi \Gamma(n_B/2 + 5/2)}. \quad (5)$$

We define the effective magnetic field  $B_{\text{eff}}$  through  $\rho_B = B_{\text{eff}}^2/(8\pi)$  and thus we get for the scale-invariant spectrum ( $n_B = -3$  [27])  $B_{\text{eff}} = B_\lambda$  for all values of  $\lambda$ . The scale-invariant case is the only case where the values of the effective and smoothed fields coincide.

We need to define the magnetic field cut-off wavenumber  $k_D$ . We assume that the cut-off scale is determined by the Alfvén wave damping scale  $k_D \sim v_A L_S$  where  $v_A$  is the Alfvén velocity and  $L_S$  the Silk damping scale [10]. Such a description is more appropriate when we are dealing with an homogeneous magnetic field and the Alfvén waves are the fluctuations  $\mathbf{B}_1(\mathbf{x})$  with respect to a background homogeneous magnetic field  $\mathbf{B}_0$  ( $|\mathbf{B}_1| \ll |\mathbf{B}_0|$ ). In the case of the stochastic magnetic field we generalize the Alfvén velocity definition, see Ref. [11], by referring to the analogy between the effective magnetic field and the homogeneous magnetic field. Assuming that the Alfvén velocity is determined by  $B_{\text{eff}}$ , a simple computation gives the expression of  $k_D$  in terms of  $B_{\text{eff}}$  [24]:

$$\frac{k_D}{1\text{Mpc}^{-1}} = 1.4 \sqrt{\frac{(2\pi)^{n_B+3} h}{\Gamma(n_B/2 + 5/2)}} \left( \frac{10^{-7}\text{G}}{B_{\text{eff}}} \right). \quad (6)$$

Here  $h$  is the Hubble constant in units of  $100 \text{ km s}^{-1} \text{ Mpc}^{-1}$ . The BBN limit on the effective magnetic field strength,  $B_{\text{eff}} \leq 8.4 \times 10^{-7} \text{ G}$  [24], gives an upper limit on the cut-off wavenumber  $k_D$ ,

$$k_D^{\text{BBN}} \geq 0.17 h^{1/2} \frac{(2\pi)^{(n_B+3)/2}}{\Gamma^{1/2}(n_B/2 + 5/2)} \text{Mpc}^{-1}. \quad (7)$$

---

<sup>5</sup> We use

$$B_j(\mathbf{k}) = \int d^3x e^{i\mathbf{k}\cdot\mathbf{x}} B_j(\mathbf{x}), \quad B_j(\mathbf{x}) = \int \frac{d^3k}{(2\pi)^3} e^{-i\mathbf{k}\cdot\mathbf{x}} B_j(\mathbf{k}), \quad (2)$$

when Fourier transforming between position and wavenumber spaces. We assume flat spatial hypersurfaces (consistent with current observational indications, [29]).

In the case of an extremely large magnetic field it is possible to have  $\lambda_D > 1$  Mpc. At this point it would seem unreasonable (unjustified) to consider a smoothing scale  $\lambda = 1$  Mpc as is conventionally done.

### III. CMB POLARIZATION PLANE ROTATION

The presence of a primordial magnetic field during recombination causes a rotation of the CMB polarization plane through the Faraday effect [15]. The rms rotation angle  $\alpha_{\text{rms}} = (\langle \alpha^2 \rangle)^{1/2}$  induced by a stochastic magnetic field with smoothed amplitude  $B_\lambda$  and spectral index  $n_B$  is given by

$$\langle \alpha^2 \rangle = \sum_l \frac{2l+1}{4\pi} C_l^\alpha, \quad (8)$$

where the rotation multipole power spectrum  $C_l^\alpha$  is [17]

$$C_l^\alpha \simeq \frac{9l(l+1)}{(4\pi)^3 q^2 \nu_0^4} \frac{B_\lambda^2}{\Gamma(n_B/2 + 3/2)} \left( \frac{\lambda}{\eta_0} \right)^{n_B+3} \int_0^{x_S} dx x^{n_B} j_l^2(x). \quad (9)$$

Here  $\eta_0$  is the conformal time today,  $\nu_0$  is the CMB photon frequency,  $q^2 = 1/137$  is the squared elementary charge in cgs units,  $j_l(x)$  is a Bessel function with argument  $x = k\eta_0$ , and  $x_S = k_S\eta_0$  where  $k_S = 2 \text{ Mpc}^{-1}$  is the Silk damping scale. In the case of an extreme magnetic field which just satisfies the BBN bound,  $k_D$  might become less than the Silk damping scale. In this case the upper limit in the integral above must be replaced by  $x_D = k_D\eta_0$ .

In terms of  $B_{\text{eff}}$ , Eq. (9) can be rewritten in the following form,

$$C_l^\alpha \simeq 1.6 \times 10^{-4} \frac{l(l+1)}{(k_D\eta_0)^{n_B+3}} \left( \frac{B_{\text{eff}}}{1 \text{ nG}} \right)^2 \left( \frac{100 \text{ GHz}}{\nu_0} \right)^4 \times \frac{n_B+3}{2} \int_0^{x_S} dx x^{n_B} j_l^2(x), \quad (10)$$

and, as a result,

$$\alpha_{\text{rms}} \simeq 0.14^\circ \left( \frac{B_{\text{eff}}}{1 \text{ nG}} \right) \left( \frac{100 \text{ GHz}}{\nu_0} \right)^2 \frac{\sqrt{n_B+3}}{(k_D\eta_0)^{(n_B+3)/2}} \times \left[ \sum_{l=0}^{\infty} (2l+1)l(l+1) \int_0^{x_S} dx x^{n_B} j_l^2(x) \right]^{1/2}. \quad (11)$$

It is of interest to compare Eq. (11) with the corresponding result, Eq. (2) of Ref. [15], derived for a homogeneous magnetic field and at frequency  $\nu_0 = 30 \text{ GHz}$ ,

$$\alpha_{\text{rms}} \simeq 1.6^\circ \left( \frac{B_0}{1 \text{ nG}} \right) \left( \frac{30 \text{ GHz}}{\nu_0} \right)^2 \quad (12)$$

Both expressions agree for  $n_B \rightarrow -3$  after accounting for  $\sum_l (2l+1)j_l^2(x) = 1$  and the fact that Bessel functions peak at  $x \sim l$  for given  $l$  (see Appendix A).

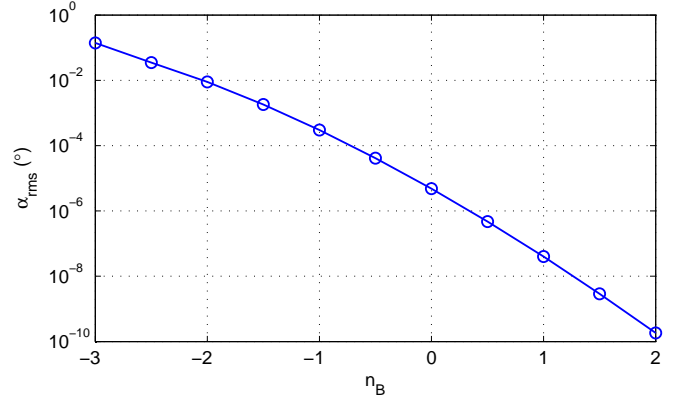


FIG. 1: Rms rotation angle  $\alpha_{\text{rms}}$  as a function of spectral index  $n_B$  for the case when  $B_{\text{eff}} = 1 \text{ nG}$  and  $\nu_0 = 100 \text{ GHz}$ . Circles correspond to the computed values.

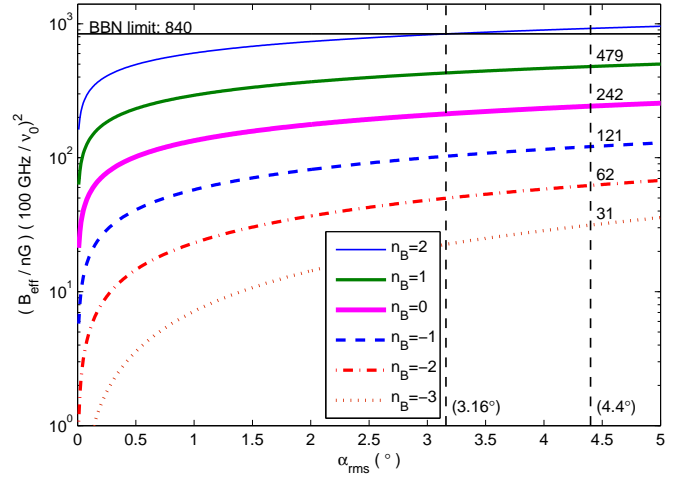


FIG. 2: Effective magnetic field limits set by the measurement of the rotation angle  $\alpha_{\text{rms}}$  for different spectral indices ( $n_B = -3, -2, -1, 0, 1, 2$ , from bottom to top). The horizontal solid line shows the upper limit set by BBN. Vertical dashed lines correspond to the angles  $\alpha_{\text{rms}} = 3.16^\circ$  that is set by the BBN limit on the effective magnetic field with spectral index  $n_B = 2$  and  $\alpha_{\text{rms}} = 4.4^\circ$  set by the WMAP 7-year data. The numerical values of the effective magnetic field constraints (in nG at 100 GHz) from the  $\alpha_{\text{rms}} = 4.4^\circ$  limit are shown on the graph for each spectral index value.

Figure 1 shows the rms rotation angle  $\alpha_{\text{rms}}$ , Eq. (11), as a function of the spectral index  $n_B$  when the effective magnetic field is normalized to be  $10^{-9} \text{ G}$ . The WMAP 7-year data limits the rms rotation angle to be less than  $4.4^\circ$  at 95% C. L. [30]. This allows us to limit the effective magnetic field as shown in Fig. 2.

#### IV. LARGE-SCALE STRUCTURE

A primordial tangled magnetic field can also induce the formation of structures in the Universe. In particular, these fields can play an important role in the formation of first structures (see, e.g. Refs. [9, 18–22, 31, 32]).

The magnetic-field-induced matter power spectrum  $P(k)$  is  $\propto k^4$  for  $n_B > -1.5$  and  $\propto k^{2n_B+7}$  for  $n_B \leq -1.5$  [19, 32]. The cut-off scale of the power spectrum is determined by the larger of the magnetic Jeans' wavenumber  $k_J$  and the thermal Jeans' wavenumber  $k_{\text{therm}}$  (for a detailed discussion, see, e.g. Ref. [22]). Here the magnetic Jeans' wavenumber is (see, e.g. Ref. [32])

$$k_J \simeq (230^{(n_B+3)/2} \times 13.8)^{2/(n_B+5)} \left( \frac{1 \text{ nG}}{B_{\text{eff}}} \right) \text{Mpc}^{-1}. \quad (13)$$

Unlike the  $\Lambda$ CDM matter power spectrum, the magnetic-field-induced matter power spectrum increases at small scales and can exceed the  $\Lambda$ CDM matter one at small scales (for a comparison of these two spectra, see, e.g. Fig. 3 of Ref. [19]). And, therefore, one of the more important contributions of the additional power induced by magnetic fields is to the formation of the first structures in the Universe (e.g. Refs. [18, 20, 21] and references therein).

In Fig. 3 we show the linear mass dispersion  $\sigma(M)$  for matter power spectra induced by a primordial magnetic field with  $B_{\text{eff}} = 6 \text{ nG}$  at  $z = 10$  for different values of  $n_B$ . Notable features of Fig. 3 are: (a) the mass dispersion on small scales is larger for a larger value of  $n_B$ ; and, (b) for  $n_B \geq -1.5$ , the mass dispersion drops more sharply at larger scales than for  $n_B \leq -1.5$ . We focus here on the mass dispersion on the smallest scales, as these scales are more relevant for the formation of the first structures in the Universe. These first structures were responsible for the reionization of the Universe at  $z \simeq 10$ . To obtain meaningful constraints on  $B_{\text{eff}}$  from the formation of first structures, we need to know how the curves shown in Fig. 3 vary as  $B_{\text{eff}}$  is changed and as the Universe evolves.

The mass dispersion  $\sigma(M, z)$  evolves with the time dependence of the growing mode of the linear density perturbations sourced by the primordial magnetic field [19, 32]. The growing mode is  $\propto a(t)$ , the scale factor, at high redshifts, the same as in the “standard”  $\Lambda$ CDM case without a magnetic field. To account for this evolution the curves corresponding to  $\sigma$  in Fig. 3 must be scaled by roughly a factor of  $\simeq 11/(1+z)$  for redshifts  $z \gg 1$ .

It can be shown that the value of  $\sigma$  at the smallest scales ( $M \simeq 10^6 M_\odot$ ) is invariant under a change in  $B_{\text{eff}}$  if the cut-off scale is determined by  $k_J$ : an increase/decrease in the value of  $B_{\text{eff}}$  is compensated by a decrease/increase in the value of  $k_J$ . However, if  $B_{\text{eff}}$  is decreased to a value at which  $k_{\text{therm}} \leq k_J$ , then the value of  $\sigma$  decreases with a decrease in  $B_{\text{eff}}$ , as the cut-off scale becomes independent of the value of  $B_{\text{eff}}$ .

It has been shown that the dissipation of magnetic fields in the post-recombination era can substantially al-

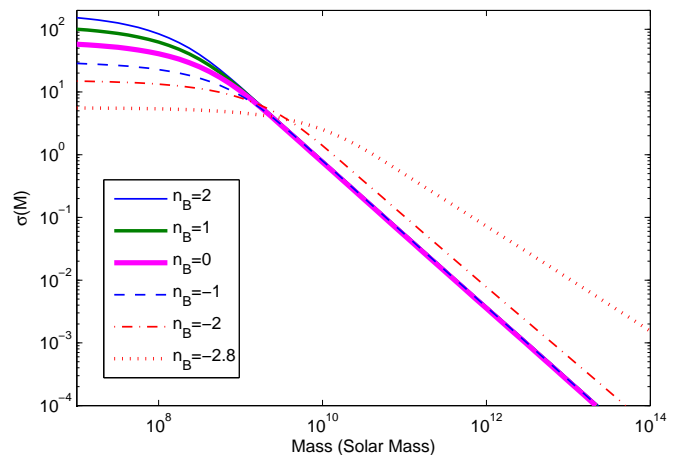


FIG. 3: The mass dispersion at  $z = 10$  for  $B_{\text{eff}} = 6 \text{ nG}$  as a function of magnetic field power spectral index  $n_B$ . From top to bottom (at the left hand side of the plot), the curves correspond to  $n_B = 2, 1, 0, -1, -2, -2.8$ .

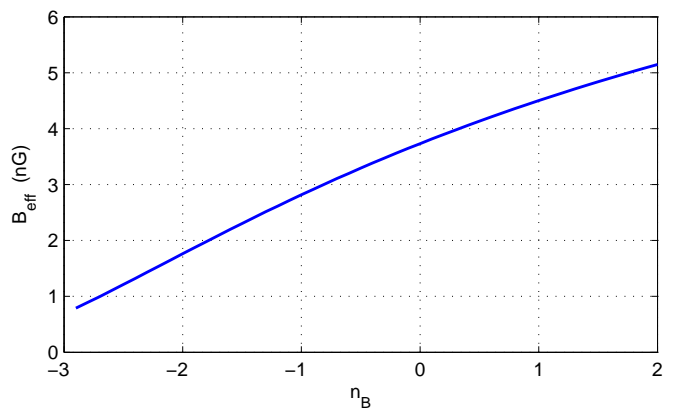


FIG. 4: Constraint on the magnetic field strength  $B_{\text{eff}}$  as a function of the power spectral index  $n_B$ .

ter the thermal and ionization history of the universe [18, 20, 22]. In particular, this dissipation raises the matter temperature and therefore the Jeans' scale in the IGM. For  $B_{\text{eff}} \geq 1 \text{ nG}$  the matter temperature rises to  $\simeq 10^4 \text{ K}$  as early as  $z \geq 100$ , [20], resulting in a steep rise in the Jeans' scale as compared to the usual case. The Jeans' wave number corresponding to this temperature is  $k_{\text{therm}} \simeq 10 \text{ Mpc}^{-1}$  (see, e.g. Fig. 4 of Ref. [22]).

WMAP results show that the Universe reionized at  $z \simeq 10$ . This reionization was caused by the non-linear collapse of the first structures, followed by star formation and the emission of UV photons from the collapsed halos. For a virialized structure in the spherical collapse model, the linear mass dispersion  $\sigma \simeq 1.7$ . This implies that the value of  $\sigma$  at the scales of interest at  $z \simeq 10$  is not expected to be much higher than 1.7. Consider the  $n_B = 2$  model in Fig. 3; the value of mass dispersion at the smallest scales is  $\simeq 100$ , which means that the first structures formed at  $z \simeq 650$  in this case (the

redshift of the collapse of first structures is  $\simeq 6.5\sigma_{\max}$ , where  $\sigma_{\max}$  is the maximum value of  $\sigma$  at  $z \simeq 10$ ), which can certainly be ruled out by the WMAP data on CMB anisotropies. A similar arguments can be used to rule out almost all the models shown in Fig. 3. Only the nearly scale-invariant models with  $n_B \simeq -3$  do not put strong constraints on the strength of the magnetic field. As argued above, the value of mass dispersion at the smallest scales to collapse is nearly independent of the magnetic field strength unless  $B_{\text{eff}}$  decreases to a value such that  $k_J = k_{\text{therm}}$ . In this case, the value of  $\sigma$  decreases below those shown in Fig. 3. We have explored a wide range of  $B_{\text{eff}}$  for the range of spectral indices shown in Fig. 3. We find that the range of acceptable values is 1–3 nG. In Fig. 4 we show the  $B_{\text{eff}}$  corresponding to  $k_J = k_{\text{therm}}$ . Notwithstanding various complications discussed above, this figure gives a rough sense of the acceptable range of  $B_{\text{eff}}$  over the entire range of  $n_B$ .

In the foregoing, we neglect the impact of the  $\Lambda$ CDM model on the process of reionization. As the density fields induced by the  $\Lambda$ CDM model and the magnetic field are uncorrelated, the matter power spectra owing to these two physical phenomena would add in quadrature. The smallest structures to collapse at  $z \simeq 10$  in the WMAP-normalized  $\Lambda$ CDM model are  $2.5\sigma$  fluctuations of the density field as opposed to the magnetic field case where  $1\sigma$  collapse is possible (Fig. 3). This means the number of collapsed halos is more abundant in the latter case. Therefore, depending on the star-formation history, if the magnetic-field-induced halo collapse made an important contribution to the reionization process, the far rarer halos from  $\Lambda$ CDM would have made a negligible impact (for further details and references see Ref. [21]).

## V. CONCLUSIONS

In this paper we study the large-scale imprints of a cosmological magnetic field, such as the rotation of the CMB polarization plane and formation of the first bound structures. We derive the corresponding limits on a primordial magnetic field energy density, expressed as limits on the effective value of the magnetic field,  $B_{\text{eff}}$ . These limits are identical to limits on the smoothed magnetic field  $B_\lambda$  (smoothed over a length scale  $\lambda$  that is conventionally taken to be 1 Mpc) only in the case of the scale-invariant magnetic field (when  $n_B = -3$ ). For a steep magnetic field with spectral index  $n_B = 2$  the difference between  $B_{\lambda=1 \text{ Mpc}}$  and  $B_{\text{eff}}$  is enormous (greater than  $10^{15}$ ). We show that using the smoothed magnetic field can result in some confusion; e.g. an extremely small smoothed magnetic field on large scales does not mean that this field cannot leave observable traces on cosmological scales.

An intergalactic magnetic field of effective value larger than 1–10 nG (with, depending on magnetic spectral index, corresponding values of  $B_{\lambda=1 \text{ Mpc}}$  in the range  $10^{-8} - 10^{-26}$  G) is ruled out by cosmological data. These limits of 1–10 nG are consistent with recent observational

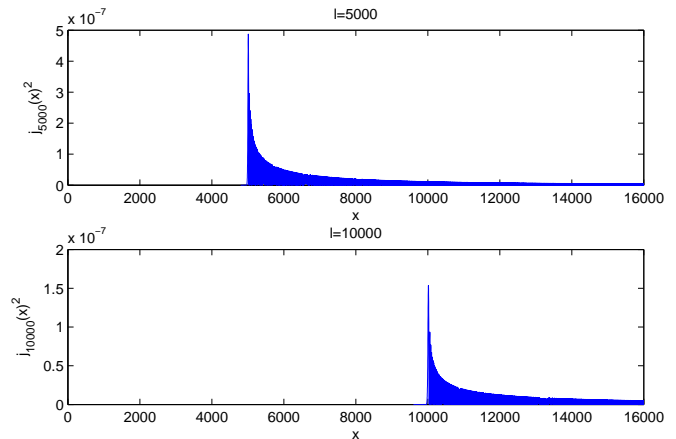


FIG. 5: The squared spherical Bessel functions  $j_l^2(x)$  for  $l = 5000$  (top) and  $l = 10000$  (bottom). Clearly  $j_l^2(x)$  peaks at  $x \approx l$ .

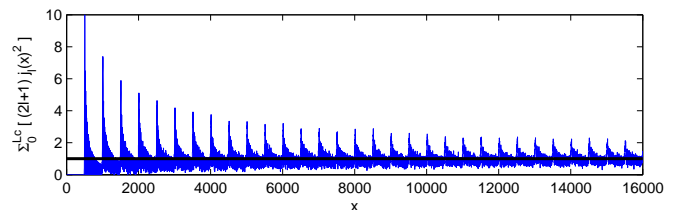


FIG. 6: The sum of the squared spherical Bessel function  $\sum_{l=0}^{l_C} (2l+1) j_l^2(x)$  for  $l_C = x_S \simeq 16000$ . The sum converges to 1 (horizontal solid line).

bounds on the intergalactic magnetic field [2–4] if the field was generated in the early Universe with spectral shape  $n_B \leq 1$ . This favors the inflationary magnetogenesis scenario.

## Acknowledgments

We acknowledge partial support from Georgian National Science Foundation grant GNSF ST08/4-422, Department of Energy grant DOE DE-FG03-99EP41043, Swiss National Science Foundation SCOPES grant no. 128040, and NASA Astrophysics Theory Program grant NNX10AC85G. T.K. acknowledges the ICTP associate membership program.

## Appendix A: Evaluating the right hand side of Eq. (11) when $n_B \rightarrow -3$

The  $\sqrt{n_B + 3}$  factor in the numerator of the right hand side of Eq. (11) is compensated by a corresponding  $1/\sqrt{n_B + 3}$  from the Bessel function integral when the spectral index  $n_B \rightarrow -3$  and so the expression for  $\alpha_{\text{rms}}$  remains finite in this limit. To establish this we use properties of the Bessel function. Recall that  $j_l^2(x)$  peaks at  $x \sim l$ , as shown in Fig. 5. This allows us to replace

the factor  $l(l+1)j_l^2(x)$  by  $x^2j_l^2(x)$  (the accuracy of this approximation is of order 15-20%). The next step is to perform the sum over  $l$ . It is obvious that there is cut-off multipole number  $l_C$  that corresponds to the cut-off wavenumber,  $l_C \sim \min(x_D, x_S)$ . Now  $j_l^2(x)$  satisfies

$$\sum_{l=0}^{\infty} (2l+1)j_l^2(x) = 1, \quad (\text{A1})$$

while we are interested in computing  $\sum_{l=0}^{l_C} (2l+1)j_l^2(x)$ . The Silk damping scale cutoff multipole number is  $l_S \simeq 16000$ , [17]. Figure 6 shows that the sum to  $l_S$  converges to 1.

- 
- [1] J. P. Vallée, *New Astron. Rev.* **48**, 763 (2004).
  - [2] A. Neronov and I. Vovk, *Science* **328**, 73 (2010).
  - [3] F. Tavecchio, et al., *Mon. Not. Roy. Astron. Soc.* **406**, L70 (2010).
  - [4] S. Ando and A. Kusenko, arXiv:1005.1924 [astro-ph.HE]; also see A. Neronov, D. V. Semikoz, P. G. Tinyakov, and I. I. Tkachev, arXiv:1006.0164 [astro-ph.HE].
  - [5] M. Giovannini, *Lect. Notes Phys.* **737**, 863 (2008).
  - [6] K. Subramanian, *Astron. Nachr.* **331**, 110 (2010).
  - [7] A. Kandus, K. E. Kunze, and C. G. Tsagas, arXiv:1007.3891 [astro-ph.CO].
  - [8] K. Subramanian and J. D. Barrow, *Phys. Rev. Lett.* **81**, 3575 (1998), *Mon. Not. Roy. Astron. Soc.* **335**, L57 (2002); T. R. Seshadri and K. Subramanian, *Phys. Rev. Lett.* **87**, 101301 (2001); L. S. Kisslinger, *Phys. Rev. D* **68**, 043516 (2003); K. Subramanian, T. R. Seshadri, and J. D. Barrow, *Mon. Not. Roy. Astron. Soc.* **344**, L31 (2003); C. Caprini, R. Durrer, and T. Kahniashvili, *Phys. Rev. D* **69**, 063006 (2004); A. Lewis, *Phys. Rev. D* **70**, 043011 (2004); M. Giovannini, *Int. J. Mod. Phys. D* **13**, 391 (2004); D. Yamazaki, K. Ichiki, T. Kajino, and G. J. Mathews, *Astrophys. J.* **646**, 719 (2006); M. Giovannini and K. E. Kunze, *Phys. Rev. D* **77**, 123001 (2008).
  - [9] K. Subramanian and J.D. Barrow, *Phys. Rev. D* **58**, 083502 (1998).
  - [10] K. Jedamzik, V. Katalinić, and A.V. Olinto, *Phys. Rev. D* **57**, 3264 (1998).
  - [11] A. Mack, T. Kahniashvili, and A. Kosowsky, *Phys. Rev. D* **65**, 123004 (2002).
  - [12] M. Giovannini and K. E. Kunze, *Phys. Rev. D* **77**, 063003 (2008); D. G. Yamazaki, K. Ichiki, T. Kajino, and G. J. Mathews, *Phys. Rev. D* **77**, 043005 (2008), **81**, 023008, 103519 (2010); M. Giovannini and K. E. Kunze, arXiv:0804.2238 [astro-ph]; D. G. Yamazaki, K. Ichiki, K. Toshitaka, and G. J. Mathews, *Mod. Phys. Lett. A* **23**, 1695 (2008); F. Finelli, F. Paci, and D. Paoletti, *Phys. Rev. D* **78**, 023510 (2008); J. R. Kristiansen and P. G. Ferreira, *Phys. Rev. D* **77**, 123004 (2008); D. Paoletti, F. Finelli, and F. Paci, *Mon. Not. R. Astron. Soc.* **396**, 523 (2009); D. Paoletti and F. Finelli, arXiv:1005.0148 [astro-ph.CO], and references therein.
  - [13] T. Kahniashvili and B. Ratra, *Phys. Rev. D* **75**, 023002 (2007).
  - [14] J. A. Adams, U. H. Danielsson, D. Grasso, and H. Rubinstein, *Phys. Lett. B* **388**, 253 (1996); R. Durrer, T. Kahniashvili, and A. Yates, *Phys. Rev. D* **58**, 123004 (1998); C. Tsagas and R. Maartens, *Phys. Rev. D* **61**, 083519 (2000); K. Jedamzik, V. Katalinic, and A. V. Olinto, *Phys. Rev. Lett.* **85**, 700 (2000); R. Durrer, P. G. Ferreira, and T. Kahniashvili, *Phys. Rev. D* **61**, 043001 (2000); G. Chen, et al., *Astrophys. J.* **611**, 655 (2004); T. R. Seshadri and K. Subramanian, *Phys. Rev. D* **72**, 023004 (2005); I. Brown and R. Crittenden, *Phys. Rev. D* **72**, 063002 (2005); H. Tashiro, N. Sugiyama, and R. Banerjee, *Phys. Rev. D* **73**, 023002 (2006); M. Giovannini, *Phys. Rev. D* **73**, 101302, **74**, 063002 (2006); K. Ichiki, et al., arXiv:astro-ph/0701329; A. Kuiroukidis, K. Kleidis, and D. B. Papadopoulos, *Int. J. Mod. Phys. A* **22**, 2197 (2007); T. Kahniashvili and B. Ratra, *Phys. Rev. D* **75**, 023002 (2007); T. Kahniashvili, G. Lavrelashvili, and B. Ratra, *Phys. Rev. D* **78**, 063012 (2008); A. Bernui and W. S. Hipolito-Ricaldi, *Mon. Not. Roy. Astron. Soc.* **389**, 1453 (2008); T. R. Seshadri and K. Subramanian, *Phys. Rev. Lett.* **103**, 081303 (2009); C. Caprini, F. Finelli, D. Paoletti, and A. Riotto, *JCAP* **0906**, 021 (2009); J. Kim and P. Naselsky, *JCAP* **0907**, 041 (2009); J. R. Shaw and A. Lewis, *Phys. Rev. D* **81**, 043517 (2010); C. Bonvin and C. Caprini, *JCAP* **1005**, 022 (2010); I. A. Brown, arXiv:1005.2982 [astro-ph.CO].
  - [15] A. Kosowsky and A. Loeb, *Astrophys. J.* **469**, 1 (1996).
  - [16] S. Sethi, *Mon. Not. R. Astron. Soc.* **342**, 962 (2003); L. Campanelli, A. D. Dolgov, M. Giannotti, and F. L. Villante, *Astrophys. J.* **616**, 1 (2004); C. Scoccola, D. Harari and S. Mollerach, *Phys. Rev. D* **70**, 063003 (2004); M. Giovannini and K. E. Kunze, *Phys. Rev. D* **78**, 023010 (2008); T. Kahniashvili, Y. Maravin, and A. Kosowsky, *Phys. Rev. D* **80**, 023009 (2009), and references therein.
  - [17] A. Kosowsky, T. Kahniashvili, G. Lavrelashvili, and B. Ratra, *Phys. Rev. D* **71**, 043006 (2005).
  - [18] R. G. Yamazaki, K. Ichiki, T. Kajino, and G. J. Mathews, *Phys. Rev. D* **78**, 123001 (2008); D. R. G. Schleicher, R. Banerjee, and R. S. Klessen, *Astrophys. J.* **692**, 236 (2009).
  - [19] R. Gopal and S. K. Sethi, *Phys. Rev. D* **72**, 103003 (2005).
  - [20] S. K. Sethi and K. Subramanian, *Mon. Not. Roy. Astron. Soc.* **356**, 778 (2005).
  - [21] S. K. Sethi and K. Subramanian, *JCAP* **0911**, 021 (2009); S. K. Sethi, Z. Haiman, and K. Pandey, arXiv:1005.2942 [astro-ph.CO].
  - [22] S. K. Sethi, B. B. Nath, and K. Subramanian, 2008, *Mon. Not. Roy. Astron. Soc.* **387**, 1589.
  - [23] C. Caprini, R. Durrer and E. Fenu, *JCAP* **0911**, 001 (2009); C. Caprini, R. Durrer and G. Servant, *JCAP* **0912**, 024 (2009).
  - [24] T. Kahniashvili, A. Tevzadze and B. Ratra, arXiv:0907.0197 (2009).
  - [25] C. Caprini and R. Durrer, *Phys. Rev. D* **65**, 023517 (2001).
  - [26] E. R. Harrison, *Mon. R. Astron. Soc.* **147**, 279 (1970); T. Vachaspati, *Phys. Lett. B* **265**, 258 (1991); A. Branden-

- burg, K. Enqvist, and P. Olesen, Phys. Rev. D **54**, 1291 (1996); J. M. Cornwall, Phys. Rev. D **56**, 6146, (1997); G. Sigl, A. V. Olinto, and K. Jedamzik, Phys. Rev. D **55**, 4582 (1997); A.-C. Davis and K. Dimopoulos, Phys. Rev. D **55**, 7398 (1997); M. Joyce and M. E. Shaposhnikov, Phys. Rev. Lett. **79**, 1193 (1997); M. Hindmarsh and A. Everett, Phys. Rev. D **58**, 103505 (1998); D. Grasso and A. Dolgov, Nucl. Phys. Proc. Suppl. **110**, 189 (2002); D. Boyanovsky, M. Simionato, and H. J. de Vega, Phys. Rev. D **67**, 023502 (2003); L. Campanelli and M. Giannotti, Phys. Rev. D **72**, 123001 (2005); T. Stevens, et al., Phys. Rev. D **77**, 023501 (2008); L. Hollenstein, C. Caprini, R. Crittenden, and R. Maartens, Phys. Rev. D **77**, 063517 (2008); A. Díaz-Gil, J. García-Bellido, M. García Pérez, and A. González-Arroyo, JHEP **0807**, 043 (2008), E. M. Henley, M. B. Johnson, and L. S. Kisslinger, Phys. Rev. D **81**, 085035 (2010); F. R. Urban and A. R. Zhitnitsky, Phys. Rev. D **82**, 043524 (2010), and references therein.
- [27] B. Ratra, Astrophys. J. Lett. **391**, L1 (1992).
- [28] K. Bamba and J. Yokoyama, Phys. Rev. D **69**, 043507 (2004); K. Bamba and M. Sasaki, JCAP **0702**, 030 (2007); J. Martin and J. Yokoyama, JCAP **0801**, 025 (2008); L. Campanelli, P. Cea, G. L. Fogli and L. Tedesco, Phys. Rev. D **77**, 123002 (2008); L. Campanelli, Int. J. Mod. Phys. D **18**, 1395 (2009); K. Bamba, N. Ohta, and S. Tsujikawa, Phys. Rev. D **78**, 043524 (2008), and references therein.
- [29] B. Ratra and M. S. Vogeley, Publ. Astron. Soc. Pac. **120**, 235 (2008).
- [30] E. Komatsu, et al., arXiv:1001.4538 [astro-ph.CO].
- [31] I. Wasserman, Astrophys. J. **224**, 337 (1978).
- [32] E. Kim, A. V. Olinto, and R. Rosner, Astrophys. J. **468**, 28 (1996).